Electron tunnelling from a quantum well in crossed electric and magnetic fields

1997 J. Phys.: Condens. Matter 9 11157

(http://iopscience.iop.org/0953-8984/9/50/017)

J. Phys.: Condens. Matter **9** (1997) 11157–11169. Printed in the UK PII: S0953-8984(97)78979-3

Electron tunnelling from a quantum well in crossed electric and magnetic fields

V Ya Demikhovskii and G A Vugalter

Department of Applied Physics and Microelectronics, Nizhny Novgorod State University, Nizhny Novgorod 603600, Russia

Received 16 May 1997

Abstract. The rate of electron tunnelling from a quantum well formed in a heterostructure and subjected to perpendicular dc electric and quantizing magnetic fields has been calculated. It has been shown that at a fixed value of magnetic field there exists a threshold value of electric field below which the tunnelling from the well is impossible. If the electric field exceeds the threshold value, the tunnelling rate increases with increasing electric field, but the dependence of the tunnelling rate on the electric field is not smooth because of new Landau levels being engaged, to which electrons can go over. It is exciting that the tunnelling rate in crossed electric and magnetic fields can be significantly higher than the rate in the presence of just the electric field. A physical explanation of this effect is given.

1. Introduction

Different quantum-mechanical phenomena exhibited by high-quality semiconductor heterostructures are currently of great interest. In particular, electron tunnelling through a rectangular barrier between two semiconductors in the presence of a magnetic field has been considered theoretically in several papers (see, for example, [1–3] and references therein). Tunnelling in single-barrier heterostructures from a quantum well into magnetoquantized interface states (these states correspond to classical electron skipping orbits) was observed in [4]. Resonant tunnelling in double-barrier heterostructures under a magnetic field applied in the plane of the tunnel barriers was investigated theoretically in [5, 6] and experimentally in [7]. The authors of [7] observed tunnelling into interfacial Landau levels of two distinct types: 'traversing' orbits and 'skipping' states. However, electron tunnelling from a quantum well in crossed dc electric and magnetic fields in the case where a barrier is absent and interfacial Landau levels are not important has not been studied either theoretically or experimentally.

The purpose of our paper is the theoretical investigation of electron tunnelling from a quantum well formed in a heterostructure and subjected to crossed dc electric and magnetic fields. The electric field is supposed to be perpendicular to the well boundaries, while the magnetic field is perpendicular to the electric one. The magnetic field is assumed to be so strong that Landau levels exist outside the well. In this case, the effective potential acting upon an electron can have two minima (see figure 1). In the absence of electron scattering, if the initial state of the electron is confined to being in the well, transitions from the well to the second minimum of the potential and back occur, and the probability of detection of the electron in the well oscillates. But electron scattering which always occurs in semiconductor heterostructures invalidates this picture. If the scattering time, τ, is much

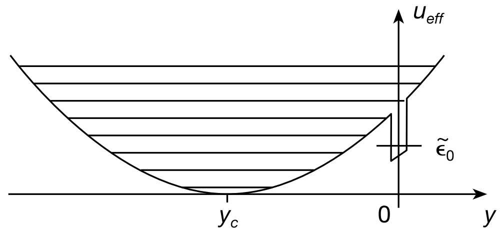

Figure 1. The effective one-dimensional potential $U_{eff}(y)$ in which the electron moves. $\tilde{\epsilon}_0$ is the energy level of the ground quasistationary state in the well. The unlabelled horizontal lines are the Landau levels ϵ_n.

less than the inverse rate of electron tunnelling from the well, w^{-1}, but much longer than the characteristic time of the electron motion in the classically forbidden region, τ_f,

$$\tau_f \ll \tau \ll w^{-1} \tag{1}$$

the return of the electron into the well is not inevitable and it is appropriate to consider tunnelling from the quantum well into Landau levels and not to consider the reverse process. Due to electron scattering outside the well, a dissipative current arises and electrons released from the well go away from the region of the well. On the other hand, due to the left-hand inequality in (1) we can neglect the electron scattering during the electron motion in the classically forbidden region and describe the electron behaviour by a Schrödinger equation. We suppose condition (1) to hold. The more exotic situation where the inelastic scattering time is much longer than the time required for tunnelling through a thin barrier between quantum Hall systems is discussed in [3]. It is shown in this paper that electrons can tunnel back and forth through the barrier and give rise to an oscillating current in the absence of external drives. It should be noticed that it was only due to electron scattering that the current flowing perpendicular to the well boundaries existed and was measured in experimental work [4, 7].

The left-hand inequality in (1) is satisfied for sufficiently pure samples at low temperatures. Moreover, we assume the heterostructure temperature to be so low that all of the electrons in the well occupy the lowest energy level and are described by the Fermi distribution function.

By applying the method used in [8], an analytical expression for the rate of electron tunnelling from the quantum well into Landau levels outside the well has been derived. In contrast to the situation considered in [4], in our case electrons tunnel not into interface states, but into bulk Landau levels. The dependence of the tunnelling rate on electric and magnetic fields has been investigated. It has been shown that at a fixed value of magnetic field there exists a threshold value of electric field below which the tunnelling from the well is impossible. If the electric field exceeds the threshold value, the tunnelling rate increases with increasing electric field, but the increase of the tunnelling rate is not smooth: whenever a transition into a new Landau level becomes possible, the derivative of the tunnelling rate with respect to the electric field tends to infinity.

We compare the tunnelling rate in crossed electric and magnetic fields with the tunnelling rate in the presence of just the electric field and show that the rate in the first case is more than the rate in the second one, if the electric field is comparatively close to a threshold value. The ratio of these two rates can be essentially greater than unity. We give a physical explanation of this effect.

In section 2 we formulate the problem and describe briefly quasistationary states of an electron in the system that we consider. In section 3 we derive an analytical expression for the tunnelling rate. In section 4 we discuss the formula obtained and give numerical results.

2. Formulation of the problem; electron quasistationary states

Let us consider a rectangular quantum well formed in a heterostructure—for example, in the structure $Al_xGa_{1-x}As$–GaAs–$Al_xGa_{1-x}As$. The well is situated between the planes $y = \pm a/2$ and subjected to crossed dc electric ($\mathcal{E}$) and magnetic ($\mathcal{H}$) fields. The fields are directed along the y- and z-axes, respectively. Let electrons occupy the ground state in the quantum well at $t = 0$. This state may be quasistationary due to the presence of the crossed fields, and we wish to establish what characteristic time is required for the two-dimensional degenerate electron gas to leave the well.

We do not take into account the electron spin because its projection on the magnetic field direction is conserved during the tunnelling process and because the additional electron energy connected with the spin is the same in the well and outside it.

We start from the stationary Schrödinger equation for an electron:

$$\left[\frac{1}{2m}\left(\hat{\boldsymbol{p}} - \frac{e}{c}\boldsymbol{A}\right)^2 - e\mathcal{E}y + U(y)\right]\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r}) \tag{2}$$

where e and m are the electron charge and effective mass, c is the velocity of light, $\hbar$ is the Planck constant and $U(y)$ is the potential energy of the well which is U_0 deep and a wide (i.e. $U(y) = -U_0$ if $|y| < a/2$, $U(y) = 0$ if $|y| > a/2$). We neglect the dependence of the effective mass on the coordinate y. It is convenient to use the magnetic vector potential in the Landau gauge $\boldsymbol{A} = (-\mathcal{H}y, 0, 0)$. In this case the Hamiltonian in equation (2) is independent of x, z and we can seek solutions of equation (2) in the form

$$\psi(\boldsymbol{r}) = \frac{1}{2\pi\hbar}\exp\left(\frac{\mathrm{i}}{\hbar}(p_x x + p_z z)\right)\varphi(y) \tag{3}$$

where p_x and p_z are the projection of the electron momentum on the x- and z-axes, respectively. Substituting equation (3) into equation (2), we obtain a one-dimensional equation:

$$\left[\frac{\hat{p}_y}{2m} + \frac{m\omega_c^2}{2}(y - y_c)^2 + U(y)\right]\varphi = \left(E - \frac{p_z^2}{2m} - v_d p_x + \frac{mv_d^2}{2}\right)\varphi. \tag{4}$$

Here

$$\omega_c = \frac{|e|\mathcal{H}}{mc} \qquad v_d = c\frac{\mathcal{E}}{\mathcal{H}} \qquad y_c = \frac{1}{m\omega_c}(p_x - mv_d) \tag{5}$$

are the cyclotron frequency, the electron drift velocity in the crossed electric and magnetic fields, and the y-coordinate of the centre of the classical electron Larmor orbit, respectively.

If one neglects the influence of the well potential on the electron motion outside the well, equation (4) yields Landau levels

$$E_{n,p_x,p_z} = \hbar\omega_c\left(n + \frac{1}{2}\right) + \frac{p_z^2}{2m} + v_d p_x - \frac{mv_d^2}{2} \qquad (n = 0, 1, 2, \ldots) \tag{6a}$$

$$\varphi_{n,p_x} = \psi_n(y - y_c) \qquad (n = 0, 1, 2, \ldots) \tag{6b}$$

where

$$\psi_n(y) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}l_H}}\exp\left(-\frac{y^2}{2l_H^2}\right)H_n\left(\frac{y}{l_H}\right) \qquad (n = 0, 1, 2, \ldots) \tag{6c}$$

are the wave functions of the harmonic oscillator [9], H_n are Hermite polynomials and $l_H = (c\hbar/|e|\mathcal{H})^{1/2}$ is the magnetic length.

Let us designate the energy of the ground state in the well in the absence of electric and magnetic fields as

$$E_0 = -\frac{\hbar^2\kappa^2}{2m} \tag{7}$$

where $1/\kappa$ is the scale of the wave-function localization outside the well. If in the well region the variation of the potential $\frac{1}{2}m\omega_c^2(y-y_c)^2$ within the space scale $a+2/\kappa$ is small compared to $|E_0|$, i.e.

$$|p_x - mv_d| \leqslant p_F + mv_d \ll \frac{\hbar\kappa}{2(2+\kappa a)}(\kappa l_H)^2 \tag{8}$$

(p_F is the Fermi momentum), one can replace the harmonic oscillator potential in equation (4) by its value at $y = 0$. After that it is obvious that there exists a bound ground state in the well. The energy of this state is

$$\tilde{E}_{0,p_x,p_z} = E_0 + \frac{(p_x - mv_d)^2}{2m} + \frac{p_z^2}{2m} + v_d p_x - \frac{mv_d^2}{2}. \tag{9a}$$

This expression can be transformed to the expected formula

$$\tilde{E}_{0,p_x,p_z} = E_0 + \frac{p_x^2 + p_z^2}{2m}. \tag{9b}$$

The wave function corresponding to energy (9a) is described approximately by the same expression as in the absence of electric and magnetic fields, namely,

$$\tilde{\varphi}_0(y) = \begin{cases} C_0\cos(ky) & |y| < a/2 \\ C_0\cos\left(\frac{1}{2}ka\right)\exp\left[-\kappa\left(|y| - \frac{1}{2}a\right)\right] & |y| > a/2 \end{cases} \tag{10}$$

where

$$C_0 = \frac{\sqrt{\kappa}}{\sqrt{1+\frac{1}{2}\kappa a}} \qquad k = \frac{1}{\hbar}\sqrt{2m(U_0 - |E_0|)} \qquad \kappa = \frac{1}{\hbar}\sqrt{2m|E_0|}. \tag{11}$$

In the general case, wave function (10) describes not a stationary state but a quasistationary one. The tunnelling rate which we would like to calculate determines the lifetime (averaged over the Fermi distribution) of electrons in this state.

Below we shall use the energies

$$\epsilon_n \equiv E_{n,p_x,p_z} - \frac{p_z^2}{2m} - v_d p_x + \frac{mv_d^2}{2} = \hbar\omega_c\left(n + \frac{1}{2}\right) \tag{12a}$$

$$\tilde{\epsilon}_0 \equiv \tilde{E}_{0,p_x,p_z} - \frac{p_z^2}{2m} - v_d p_x + \frac{mv_d^2}{2} = E_0 + \frac{(p_x - mv_d)^2}{2m} \tag{12b}$$

instead of energies (6a), (9a). The electron energy levels and the effective one-dimensional potential

$$U_{eff}(y) = \frac{1}{2}m\omega_c^2(y-y_c)^2 + U(y)$$

in which the electron moves are shown in figure 1.

3. The tunnelling rate

Let us consider the process of tunnelling from the well. The wave function $\psi(\boldsymbol{r}, t)$ of an electron obeys the Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \left[\frac{1}{2m}\left(\hat{\boldsymbol{p}} - \frac{e}{c}\boldsymbol{A}\right)^2 - e\mathcal{E}y + U(y)\right]\psi \tag{13a}$$

with the initial condition

$$\psi(\boldsymbol{r}, 0) = \frac{1}{2\pi\hbar}\exp\left(\frac{\mathrm{i}}{\hbar}(p_x x + p_z z)\right)\tilde{\varphi}_0(y) \tag{13b}$$

where $\tilde{\varphi}_0(y)$ is the wave function of the ground state in the well (see equation (10)). The problem can be reduced to a spatially one-dimensional case by making the substitution

$$\psi(\boldsymbol{r}, t) = \frac{1}{2\pi\hbar}\exp\left[\frac{\mathrm{i}}{\hbar}(p_x x + p_z z) - \frac{\mathrm{i}}{\hbar}\left(\frac{p_z^2}{2m} + v_d p_x - \frac{m v_d^2}{2}\right)t\right]\varphi(y, t). \tag{14}$$

The equation for the function $\varphi(y, t)$ reads

$$\mathrm{i}\hbar\frac{\partial\varphi}{\partial t} = \left[\frac{\hat{p}_y^2}{2m} + \frac{1}{2}m\omega_c^2(y - y_c)^2 + U(y)\right]\varphi. \tag{15}$$

Substitution (14) enables us to eliminate the term $(1/2m)p_z^2 + v_d p_x - \frac{1}{2}m v_d^2$ from expressions (6*a*), (9*a*) and to arrive at energies (12*a*), (12*b*). As is clear from figure 1, at fixed values of p_x, p_z the electron can leave the well if the energy level of the initial state coincides with one of the Landau levels, i.e.

$$E_0 + \frac{1}{2m}(p_x - m v_d)^2 = \hbar\omega_c\left(n + \frac{1}{2}\right). \tag{16}$$

From equation (16) we find

$$|p_x - m v_d| = \sqrt{2m\left(|E_0| + \hbar\omega_c\left(n + \frac{1}{2}\right)\right)}. \tag{17}$$

According to inequality (8), the left-hand side of equation (17) should be much less than $\sqrt{2m|E_0|^3}[\hbar\omega_c(1 + \frac{1}{2}\kappa a)]^{-1}$. Therefore, in the approximation of a comparatively deep well (or comparatively weak fields), the electron can leave the well if the condition

$$\left(\frac{|E_0|}{\hbar\omega_c}\right)^{3/2}\frac{1}{2(1 + \frac{1}{2}\kappa a)} \gg \sqrt{\frac{|E_0|}{\hbar\omega_c} + n + \frac{1}{2}} \tag{18a}$$

holds. This is possible if, at least,

$$\frac{|E_0|}{\hbar\omega_c} = \frac{1}{2}(\kappa l_H)^2 \gg 1. \tag{18b}$$

This inequality is necessary but may be insufficient to ensure that condition (18*a*) is satisfied. If the number of the Landau level to which the tunnelling is proceeding is not very large (namely, $n \lesssim |E_0|/\hbar\omega_c$), condition (18*a*) can be replaced by

$$\frac{|E_0|}{\hbar\omega_c} \gg 2\left(1 + \frac{1}{2}\kappa a\right). \tag{18c}$$

Following the method described in [8], let us rewrite equation (15) in the form

$$\mathrm{i}\hbar\frac{\partial\varphi}{\partial t} - \left[\frac{\hat{p}_y^2}{2m} + \frac{1}{2}m\omega_c^2(y - y_c)^2\right]\varphi = U(y)\varphi. \tag{19}$$

Then let us introduce the Green function $G(y, y', t-t')$, which is the solution of the equation

$$\mathrm{i}\hbar\frac{\partial G}{\partial t} - \left[\frac{\hat{p}_y^2}{2m} + \frac{1}{2}m\omega_c^2(y-y_c)^2\right]G = \mathrm{i}\hbar\delta(t-t')\delta(y-y') \tag{20}$$

and which is identically equal to zero for $t < t'$. It can be shown that

$$G(y, y', t-t') = \Theta(t-t')\sum_{n=0}^{\infty}\psi_n(y-y_c)\psi_n(y'-y_c)\mathrm{e}^{-\mathrm{i}\omega_c(n+1/2)(t-t')}. \tag{21}$$

Here $\Theta(t-t')$ is a step function ($\Theta(t-t') = 1$ if $t > t'$ and $\Theta(t-t') = 0$ if $t < t'$). The Green function enables us to rewrite equation (19) and its initial condition in the integral form

$$\varphi(y, t) = \int_{-\infty}^{\infty} G(y, y', t)\tilde{\varphi}_0(y')\,\mathrm{d}y' + \frac{1}{\mathrm{i}\hbar}\int_{-\infty}^{\infty}\mathrm{d}y'\int_0^t \mathrm{d}t'\; G(y, y', t-t')U(y')\varphi(y', t'). \tag{22}$$

The first term in the right-hand side of equation (22) describes the transitions of the electron from the initial state to Landau levels when the quantum well is absent. The probabilities of these transitions do not depend on time (in contrast to the probabilities described by the second term in the right-hand side of equation (22)). Our estimates show that for sufficiently large t ($t \gg \hbar/|E_0|$) we can neglect the contribution of the first term in the right-hand side of equation (22) in comparison with the contribution of the second term. Below we shall focus on investigation of the latter.

If the time t that we consider is significantly less than the inverse tunnelling rate w^{-1}, i.e.

$$\hbar/|E_0| \ll t \ll w^{-1} \tag{23}$$

(w will be given below), we can substitute the unperturbed wave function of the initial state

$$\varphi(y', t') \simeq \tilde{\varphi}_0(y')\exp\left(-\frac{\mathrm{i}}{\hbar}\tilde{\epsilon}_0 t'\right)$$

into the right-hand side of equation (22). Using the Green function (21), we find

$$\begin{aligned}\varphi(y, t) \simeq \frac{U_0}{\hbar}\sum_{n=0}^{\infty}\psi_n(y-y_c)\mathrm{e}^{-\mathrm{i}\omega_c(n+1/2)t}\;\frac{\mathrm{e}^{\mathrm{i}(\omega_c(n+1/2)-\tilde{\epsilon}_0/\hbar)t}-1}{\omega_c(n+\frac{1}{2})-\tilde{\epsilon}_0/\hbar}\\ \times\int_{-a/2}^{a/2}\tilde{\varphi}_0(y')\psi_n(y'-y_c)\,\mathrm{d}y'.\end{aligned} \tag{24}$$

The integral contained in equation (24) can be solved analytically if the well is not wide, namely, if the well width satisfies the conditions

$$a < l_H \tag{25a}$$

$$n\frac{a}{2} \ll |y_c|. \tag{25b}$$

Taking into account expressions (5), (17), we rewrite inequality (25b) in the form

$$\frac{n}{2\sqrt{2}}\frac{a}{l_H} \ll \sqrt{\frac{|E_0|}{\hbar\omega_c} + n + \frac{1}{2}}. \tag{25c}$$

Under conditions (25a), (25c) we can approximate the function $\psi_n(y' - y_c)$ in equation (24) by $\psi_n(-y_c)\exp(y_c y'/l_H^2)$ and carry out the integration over y'. As a result, we have

$$\varphi(y,t) \simeq \sum_{n=0}^{\infty} \psi_n(y-y_c)\mathrm{e}^{-\mathrm{i}\omega_c(n+1/2)t} \left\{ \frac{U_0}{\hbar} \frac{\mathrm{e}^{\mathrm{i}(\omega_c(n+1/2)-\tilde{\epsilon}_0/\hbar)t} - 1}{\omega_c(n+\frac{1}{2}) - \tilde{\epsilon}_0/\hbar} \times \sqrt{\frac{\kappa}{1+\frac{1}{2}\kappa a}}\psi_n(-y_c)\left[\frac{\sin(k+\mathrm{i}y_c/l_H^2)a/2}{k+\mathrm{i}y_c/l_H^2} + \frac{\sin(k-\mathrm{i}y_c/l_H^2)a/2}{k-\mathrm{i}y_c/l_H^2}\right]\right\}. \tag{26}$$

The expression in the braces is simply the probability amplitude (let us designate it as $a_n(t)$) of the electron transition to the nth Landau level during the time t. In accordance with the transcendental equation governing the electron spectrum in the quantum well, the following relation holds:

$$\begin{aligned}\frac{U_0}{\hbar}&\left[\frac{\sin(k+\mathrm{i}y_c/l_H^2)a/2}{k+\mathrm{i}y_c/l_H^2} + \frac{\sin(k-\mathrm{i}y_c/l_H^2)a/2}{k-\mathrm{i}y_c/l_H^2}\right]\\ &= \sqrt{\frac{2}{m}}\frac{\sqrt{(U_0-|E_0|)U_0}}{U_0-|E_0|+(1/2m)(p_x-mv_d)^2}\\ &\quad\times\left[\sqrt{|E_0|}\cosh{(p_x-mv_d)}\frac{a}{2\hbar} + \frac{p_x-mv_d}{\sqrt{2m}}\sinh{(p_x-mv_d)}\frac{a}{2\hbar}\right].\end{aligned} \tag{27}$$

Hence, the probability of the electron transition from the well to the nth Landau level occurring during the time t equals

$$\begin{aligned}|a_n(t)|^2 &= \frac{2\kappa(U_0-|E_0|)U_0|E_0|}{m(1+\frac{1}{2}\kappa a)(U_0-|E_0|+(1/2m)(p_x-mv_d)^2)^2}\left[\psi_n\left(\frac{mv_d-p_x}{m\omega_c}\right)\right]^2\\ &\quad\times\left[\cosh{(p_x-mv_d)}\frac{a}{2\hbar} + \frac{p_x-mv_d}{\sqrt{2m|E_0|}}\sinh{(p_x-mv_d)}\frac{a}{2\hbar}\right]^2\\ &\quad\times\frac{4\sin^2(\omega_c(n+\frac{1}{2})-\tilde{\epsilon}_0/\hbar)t/2}{(\omega_c(n+\frac{1}{2})-\tilde{\epsilon}_0/\hbar)^2}.\end{aligned} \tag{28}$$

Now we can calculate the number $N(t)$ of electrons that have left the well during the time t. As mentioned above, we supposed the heterostructure temperature to be sufficiently

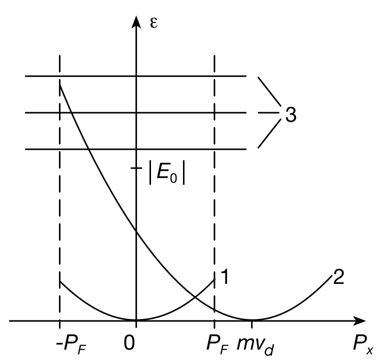

Figure 2. The parabolas $\varepsilon = (1/2m)(p_x - mv_d)^2$ for $mv_d = 0$ (curve 1) and $mv_d > p_F$ (curve 2), and horizontal lines $\varepsilon = |E_0| + \hbar\omega_c(n+\frac{1}{2}), n = 0, 1, 2, \ldots$ (curves 3), in the $p_x\varepsilon$-plane. When depicting curve 1, we took into account that $-|E_0| + (1/2m)p_F^2 < 0$.

low that we could assume the electron distribution function $f(p_x, p_z)$ at $t = 0$ to be the two-dimensional Fermi function, i.e. $f(p_x, p_z) = \Theta(p_F - \sqrt{p_x^2 + p_z^2})$ where Θ is a step function. In this case

$$\begin{aligned}
N(t) = \int\int \frac{S\,\mathrm{d}p_x\,\mathrm{d}p_z}{(2\pi\hbar)^2}\; f(p_x, p_z)\sum_{n=0}^{\infty}|a_n(t)|^2 \\
= \frac{S\kappa(U_0 - |E_0|)U_0|E_0|}{\pi\hbar^2 m(1+\frac{1}{2}\kappa a)}\int_{-p_F}^{p_F}\mathrm{d}p_x\,\sqrt{p_F^2 - p_x^2} \\
\times \sum_{n=0}^{\infty}\frac{1}{(U_0 - |E_0| + (1/2m)(p_x - mv_d)^2)^2}\left[\psi_n\left(\frac{mv_d - p_x}{m\omega_c}\right)\right]^2 \\
\times\left[\cosh\,(p_x - mv_d)\frac{a}{2\hbar} + \frac{p_x - mv_d}{\sqrt{2m|E_0|}}\sinh\,(p_x - mv_d)\frac{a}{2\hbar}\right]^2 \\
\times\left\{\frac{4\sin^2(\omega_c(n+\frac{1}{2}) + |E_0|/\hbar - (p_x - mv_d)^2/2m\hbar)t/2}{\pi(\omega_c(n+\frac{1}{2}) + |E_0|/\hbar - (p_x - mv_d)^2/2m\hbar)^2}\right\}.
\end{aligned} \tag{29}$$

Here S is the area of the heterostructure in the x–z plane. As is well known [9], for sufficiently large t the function $\sin^2(\alpha t)/\pi\alpha^2$ can be considered as $t\delta(\alpha)$ where $\delta(\alpha)$ is a delta function. In our case, for large t,

$$t \gg \hbar/\sqrt{\hbar\omega_c|E_0|} \qquad \hbar/\sqrt{\varepsilon_F|E_0|} \tag{30}$$

where $\varepsilon_F = p_F^2/2m$ is the Fermi energy, we can replace the expression in the braces (see equation (29)) by

$$t\delta\left[\frac{1}{2\hbar}\left(\hbar\omega_c\left(n+\frac{1}{2}\right) + |E_0| - \frac{1}{2m}(p_x - mv_d)^2\right)\right].$$

After that the integration over p_x can be easily carried out. Due to the delta function, only the points of intersection of the parabola $\varepsilon = (1/2m)(p_x - mv_d)^2$ and the horizontal lines $\varepsilon = |E_0| + \hbar\omega_c(n+\frac{1}{2}),\ n = 0, 1, 2, \ldots$ (see figure 2), contribute to the integral (if these points of intersection exist in the interval $-p_F < p_x < p_F$). Figure 2 shows that the points of intersection are situated to the left of the point $p_x = mv_d$. It should be noticed that the number of electrons that have left the well during the time t is proportional to this time. Therefore we can introduce the electron current J from the well, equal to the number of electrons that have left per unit of time, i.e. $J = N(t)/t$. We do not give an expression for J, but at once go over to the tunnelling rate defined as $w = J/N_0$. Here

$$N_0 = \int\int\frac{S\,\mathrm{d}p_x\,\mathrm{d}p_z}{(2\pi\hbar)^2}\; f(p_x, p_z) = \frac{Sp_F^2}{4\pi\hbar^2}$$

is the initial number of electrons in the quantum well. As follows from equations (29) and (6*c*),

$$\begin{aligned}
w = \frac{8(U_0 - |E_0|)U_0|E_0|^{3/2}}{(1+\frac{1}{2}\kappa a)\hbar\varepsilon_F^{1/2}}\sum_{n_{min}}^{n_{max}}\frac{\sqrt{1 - (mv_d/p_F - (\hbar/p_F l_H)\sqrt{2|E_0|/\hbar\omega_c + 2n + 1})^2}}{\sqrt{2|E_0|/\hbar\omega_c + 2n + 1}\;(U_0 + \hbar\omega_c(n+\frac{1}{2}))^2} \\
\times\frac{1}{2^n n!\sqrt{\pi}}\,\mathrm{e}^{-(2|E_0|/\hbar\omega_c + 2n + 1)}H_n^2\left(\sqrt{2\frac{|E_0|}{\hbar\omega_c} + 2n + 1}\right)
\end{aligned}$$

$$\times\left[\cosh\left(\frac{\kappa a}{2}\sqrt{1+\frac{2n+1}{\kappa^2 l_H^2}}\right)+\sqrt{1+\frac{2n+1}{\kappa^2 l_H^2}}\sinh\left(\frac{\kappa a}{2}\sqrt{1+\frac{2n+1}{\kappa^2 l_H^2}}\right)\right]^2. \tag{31}$$

Here n_{max} is the integral part of the number

$$N_{max}=\frac{1}{\hbar\omega_c}\left[\frac{1}{2m}(p_F+mv_d)^2-|E_0|-\frac{1}{2}\hbar\omega_c\right] \tag{32a}$$

and n_{min} is the integral part of the number

$$N_{min}=\frac{1}{\hbar\omega_c}\left[\frac{1}{2m}(-p_F+mv_d)^2-|E_0|-\frac{1}{2}\hbar\omega_c\right]+1 \tag{32b}$$

if $N_{min}>1$, and $n_{min}=0$ if $N_{min}<1$. If $N_{max}<0$ it is impossible for there to be an electron current from the well and the tunnelling rate is equal to zero because the energies of electrons in the well are less than the energies of electrons outside the well.

We emphasize that equation (31) is valid if

$$w\ll\sqrt{\hbar\omega_c|E_0|}/\hbar \qquad \sqrt{\varepsilon_F|E_0|}/\hbar \tag{32c}$$

(see inequalities (23), (30)) and n_{max} satisfies condition (25c). The restriction on the quantity n_{max} means that, at a fixed magnetic field, formula (31) describes the ionization rate in an interval of $\mathcal{E}$, bounded above.

4. Discussion and numerical results

Let us analyse equations (31) and (32). As we have mentioned, the electron current from the well (and the tunnelling rate, too) is not equal to zero if $N_{max}>0$, i.e. the inequality

$$\mathcal{E}>\mathcal{E}_{th}\equiv\frac{1}{mc}\mathcal{H}\left(\sqrt{2m\left(|E_0|+\frac{1}{2}\hbar\omega_c\right)}-p_F\right) \tag{33}$$

is satisfied. Thus, if at a fixed magnetic field the value of the electric field is less than the threshold value $\mathcal{E}_{th}$, tunnelling from the well is impossible. From condition (33) we can also conclude that at a fixed value of the electric field, tunnelling from the well is impossible if the magnetic field exceeds some value $\mathcal{H}_{th}$. Taking into account inequality (18b), we can write, approximately,

$$\mathcal{H}_{th}\simeq\frac{mc}{\sqrt{2m|E_0|}-p_F}\mathcal{E}. \tag{34}$$

Let the electric field increase and the magnetic field be fixed. Every time the electric field exceeds the value

$$\mathcal{E}_n=\frac{1}{mc}\mathcal{H}\left(\sqrt{2m\left(|E_0|+\hbar\omega_c\left(n+\frac{1}{2}\right)\right)}-p_F\right) \qquad n=0,1,2,\ldots \tag{35}$$

(obviously, $\mathcal{E}_0=\mathcal{E}_{th}$), the transition to the nth Landau level becomes possible (in addition to the transitions to the Landau levels with previous numbers lower than n for $\mathcal{E}<\mathcal{E}_n$). As follows from equation (31), the derivative of the tunnelling rate with respect to $\mathcal{E}$ equals infinity at $\mathcal{E}=\mathcal{E}_n+0$. Similarly, when the electric field exceeds the value

$$\tilde{\mathcal{E}}_n=\frac{1}{mc}\mathcal{H}\left(\sqrt{2m\left(|E_0|+\hbar\omega_c\left(n+\frac{1}{2}\right)\right)}+p_F\right) \qquad n=0,1,2,\ldots \tag{36}$$

the transition to the nth Landau level becomes impossible, and the quantity $\mathrm{d}w/\mathrm{d}\mathcal{E}$ equals minus infinity at $\mathcal{E} = \tilde{\mathcal{E}}_n - 0$. However, for typical parameters of the heterostructure (see below) the quantities $\tilde{\mathcal{E}}_n$ $(n = 0, 1, 2, \ldots)$ are large and do not belong to the interval of $\mathcal{E}$ for which formula (31) is applicable.

Our consideration is valid under condition (18b). If only Landau levels labelled with small numbers are involved in the tunnelling process (i.e. $n_{max} \ll |E_0|/\hbar\omega_c$), equation (31) can be simplified. As a result, we have

$$w \simeq \frac{4\sqrt{2}(U_0 - |E_0|)|E_0|\sqrt{\hbar\omega_c}}{(1 + \frac{1}{2}\kappa a)\hbar U_0\sqrt{\varepsilon_F}} \mathrm{e}^{\kappa a} \sqrt{1 - \left(\frac{mv_d - \hbar\kappa}{p_F}\right)^2} \times \sum_{n_{min}}^{n_{max}} \frac{1}{2^n n!\sqrt{\pi}} \mathrm{e}^{-(2|E_0|/\hbar\omega_c + 2n+1)} H_n^2\left(\sqrt{2\frac{|E_0|}{\hbar\omega_c} + 2n + 1}\right). \tag{37}$$

It is interesting to compare the tunnelling rate found above with that in the absence of a magnetic field. According to [8],

$$w(\mathcal{H} = 0) \simeq \frac{2(U_0 - |E_0|)|E_0|}{(1 + \frac{1}{2}\kappa a)\hbar U_0} \exp\left(\kappa a - \frac{4|E_0|\kappa}{3|e|\mathcal{E}}\right). \tag{38}$$

Hence, the ratio of the tunnelling rates in the presence and in the absence of a magnetic field is

$$\begin{aligned} \frac{w}{w(\mathcal{H} = 0)} \simeq {} & \frac{4U_0^2\sqrt{|E_0|}}{\sqrt{\varepsilon_F}} \exp\left(\frac{4|E_0|\kappa}{3|e|\mathcal{E}} - \kappa a\right) \\ & \times \sum_{n_{min}}^{n_{max}} \left\{ \sqrt{1 - \left(\frac{mv_d}{p_F} - \frac{\hbar}{p_F l_H}\sqrt{2\frac{|E_0|}{\hbar\omega_c} + 2n + 1}\right)^2} \right. \\ & \times \left. \left[\sqrt{2\frac{|E_0|}{\hbar\omega_c} + 2n + 1}\left(U_0 + \hbar\omega_c\left(n + \frac{1}{2}\right)\right)^2\right]^{-1} \right\} \\ & \times \frac{1}{2^n n!\sqrt{\pi}} \mathrm{e}^{-(2|E_0|/\hbar\omega_c + 2n+1)} H_n^2\left(\sqrt{2\frac{|E_0|}{\hbar\omega_c} + 2n + 1}\right) \\ & \times \left[\cosh\left(\frac{\kappa a}{2}\sqrt{1 + \frac{2n+1}{\kappa^2 l_H^2}}\right) + \sqrt{1 + \frac{2n+1}{\kappa^2 l_H^2}} \sinh\left(\frac{\kappa a}{2}\sqrt{1 + \frac{2n+1}{\kappa^2 l_H^2}}\right)\right]^2. \end{aligned} \tag{39}$$

The dependence of the tunnelling rate on the electric field at two fixed values of the magnetic field is plotted in figure 3. We have chosen the heterostructure parameters as follows: $U_0 = 0.3$ eV, $a = 42.3$ Å, $m = 0.07m_0$ (m_0 is the free-electron mass). In this case the energy of the ground state in the well is $E_0 = -0.2$ eV. We suppose the sheet density of electrons in the well to be $N_0/S = 10^{12}$ cm^{-2}. As one can see, the stronger the electric field, the higher the tunnelling rate. But the dependence $w(\mathcal{E})$ is not smooth. As we have mentioned, the derivative of w with respect to $\mathcal{E}$ tends to infinity at the points $\mathcal{E} = \mathcal{E}_n$ (see equation (35)).

In accordance with inequality (1), the numerical results represented in figure 3 are valid if $\tau \ll 10^{-5}$ s for $\mathcal{H} = 80$ kOe and if $\tau \ll 10^{-8}$ s for $\mathcal{H} = 100$ kOe. These inequalities are satisfied for real heterostructures.

The dependence of ratio (39) on the electric field is depicted in figure 4. The heterostructure parameters are the same as in figure 3. Figure 4 shows that the ratio $w/w(\mathcal{H} = 0)$

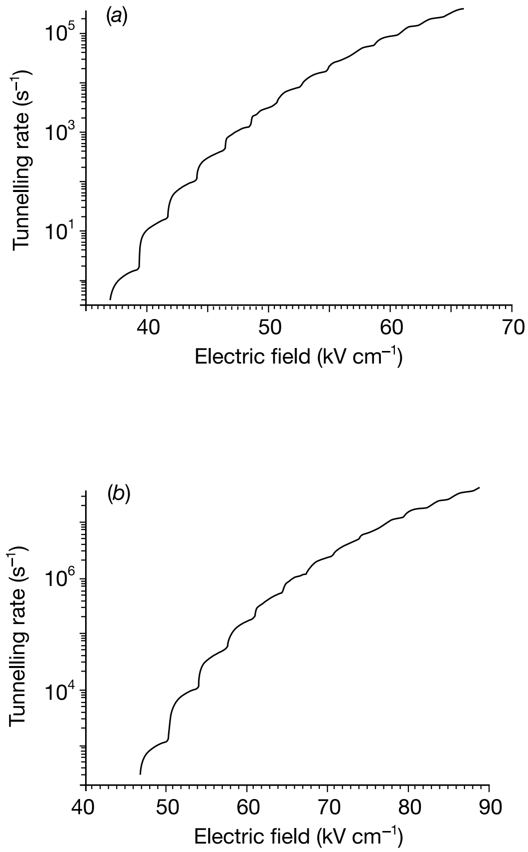

Figure 3. The dependence of the tunnelling rate on the electric field. The heterostructure parameters are as follows: $U_0 = 0.3$ eV, $a = 42.3$ Å, $m = 0.07m_0$ (m_0 is the free-electron mass) and the electron sheet density in the well is 10^{12} cm^{-2}. The magnetic field equals 80 kOe (a) and 100 kOe (b).

is a nonmonotonic function of the electric field. On the whole, the ratio decreases due to a strong exponential dependence of $w(\mathcal{H} = 0)$ on $\mathcal{E}$. But near the points $\mathcal{E} = \mathcal{E}_n$ (see equation (35)) where the tunnelling rate in the presence of a magnetic field is a sharply increasing function of the electric field, the ratio $w/w(\mathcal{H} = 0)$ increases. Figure 4 also shows that the ratio $w/w(\mathcal{H} = 0)$ can markedly exceed unity. We interpret this fact as follows. In the absence of a magnetic field an electron tunnels from the well through a triangular barrier the height of which is $|E_0|$ and the width of which is determined by the electric field and equals $b_1 = |E_0|/|e|\mathcal{E}$. In the presence of a magnetic field the electron tunnels through an approximately triangular barrier of the same height and of the width b_2 determined by the force $m\omega_c^2|y_c|$. If the electric field is just a little larger than the threshold value determined by equation (33), only the tunnelling process to the lowest Landau level is possible. In this case, we have

$$\frac{b_2}{b_1} \simeq 1 - \frac{1}{\sqrt{2m(|E_0| + \frac{1}{2}\hbar\omega_c)}}\left(p_F - \frac{|e|(\mathcal{E} - \mathcal{E}_{th})}{\omega_c}\right). \qquad (40)$$

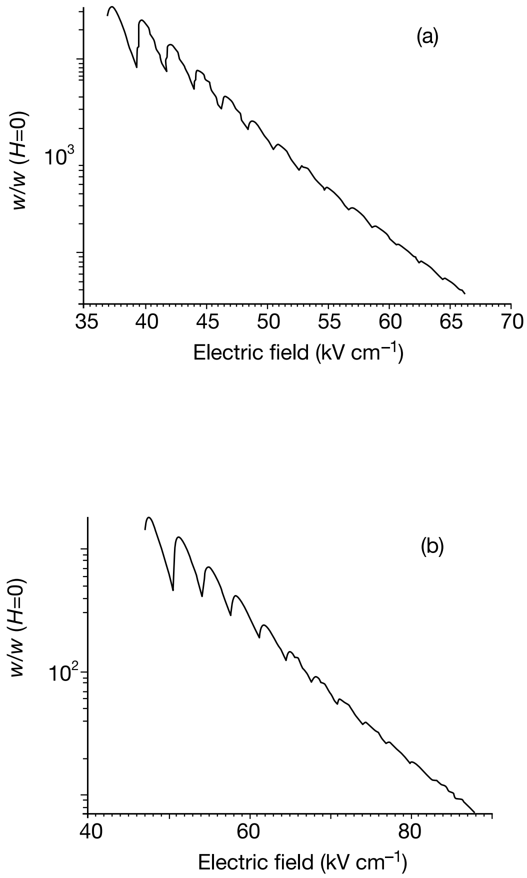

Figure 4. The ratio $w/w(\mathcal{H} = 0)$ versus the electric field for two values of magnetic field ($\mathcal{H} = 80$ kOe (a), $\mathcal{H} = 100$ kOe (b)). The heterostructure parameters are the same as in figure 3.

One can see that the barrier width in the presence of a magnetic field is smaller than that in the case where the magnetic field is absent. The ratio of the barrier transparencies in the presence and in the absence of a magnetic field is an exponential function of the quantity

$$\frac{4|E_0|\kappa}{3|e|\mathcal{E}}\left(1 - \frac{b_2}{b_1}\right) \simeq \frac{4|E_0|\kappa}{3|e|\mathcal{E}} \frac{1}{\sqrt{2m(|E_0| + \frac{1}{2}\hbar\omega_c)}}\left(p_F - \frac{|e|(\mathcal{E} - \mathcal{E}_{th})}{\omega_c}\right). \tag{41}$$

This quantity is positive and decreases with the increase of the electric field. Hence the ratio of the barrier transparencies in the presence and in the absence of a magnetic field is much larger than unity and decreases with the increase of the electric field. We can state the same for the tunnelling into other Landau levels labelled with comparatively small numbers, which occurs at larger electric fields. Since the behaviour of the tunnelling rate, on the whole, is determined by the barrier transparency, for comparatively weak electric fields the ratio $w/w(\mathcal{H} = 0)$ is much greater than unity and decreases, on the whole, with the increase of the electric field.

5. Conclusions

We have investigated electron tunnelling from a rectangular quantum well placed in crossed dc electric and magnetic fields. Electrons are supposed to experience scattering (see inequality (1)), due to which it is appropriate to speak about the tunnelling from the well into bulk Landau levels. We have shown that at a fixed value of the magnetic field there exists a threshold value of the electric field below which tunnelling from the well is impossible. We can also say that at a fixed value of the electric field there exists a threshold value of the magnetic field above which the tunnelling is impossible.

At a fixed value of the magnetic field the dependence of the tunnelling rate on the electric field is an increasing function. This dependence is not smooth because of new Landau levels becoming engaged, to which electrons can go over from the well.

The ratio of the tunnelling rates in two cases, namely, in crossed electric and magnetic fields and in the presence of just the electric field, decreases, on the whole, with the increase of the electric field at a fixed magnetic field. This ratio significantly exceeds unity if the electric field is comparatively weak because in the two cases an electron tunnels through barriers of the same height, but the width of the barrier in the presence of a magnetic field is less than that in the absence of magnetic fields.

Numerical estimates of the tunnelling rate are given.

Acknowledgments

This work was supported by the High School Committee of Russia (Grant No 95-0-5.5-63) and the Russian Foundation for Basic Research (Grant No 96-02-18067a).

References

[1] D'yakonov M I and Raikh M E 1985 *Zh. Eksp. Teor. Fiz.* **88** 1898 (Engl. Transl. 1985 *Sov. Phys.–JETP*)
[2] Brey L, Platero G and Tejedor C 1988 *Phys. Rev.* B **38** 9649
[3] Ho T-L 1994 *Phys. Rev.* B **50** 4524
[4] Snell B R, Chan K S, Sheard F W, Eaves L, Toombs G A, Maude D K, Portal J C, Bass S J, Claxton P A, Hill G and Pate M A 1987 *Phys. Rev. Lett.* **59** 2806
[5] Platero G, Brey L and Tejedor C 1989 *Phys. Rev.* B **40** 8548
[6] Hung K-M and Wu G Y 1992 *Phys. Rev.* B **45** 3461
[7] Leadbeater M L, Alves E S, Eaves L, Henini M, Hughes O H, Celeste A, Portal J C, Hill G and Pate M A 1989 *J. Phys.: Condens. Matter* **1** 4865
[8] Demikhovskii V Ya and Vugalter G A 1996 *J. Phys.: Condens. Matter* **8** 2585
[9] Landau L D and Lifshitz E M 1977 *Quantum Mechanics* 3rd edn (New York: Pergamon)